INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2021-22
Statistics - II, Backpaper Examination
Answer all questions **Maximum Marks: 50** **Time: 3 Hours**

1. Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x_i|\theta) = \begin{cases} \frac{1}{2i\theta} & \text{if } -i(\theta-1) < x_i < i(\theta+1); \\ 0 & \text{otherwise,} \end{cases}$$

$1 \leq i \leq n$, where $\theta > 0$.
(a) Find a two-dimensional sufficient statistic for θ.
(b) Find the maximum likelihood estimator of θ. **[15]**

2. Consider a random sample from $N(0, \sigma^2)$.
(a) Find the UMVUE of σ.
(b) Show that the UMVUE of σ is a consistent estimator.
(c) Find the asymptotic distribution of the UMVUE of σ. **[12]**

3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from Poisson(λ). Consider testing

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.
(b) Derive the UMP test of level α. **[8]**

4. A large shipment of parts is received, out of which 5 are tested for defects. Let X denote the number of defective parts in the sample, and θ be the proportion of defective parts in the population. From past shipments it is known that θ has a Beta(1, 9) distribution.
(a) Find the HPD estimate of θ if $X = 0$ is observed.
(b) Find a 95% credible set for θ if $X = 0$ is observed. **[15]**